School of Computational and Mathematical Sciences (Programme of Mathematics and Applied Mathematics)

Postgraduate Programmes

Modules

Honours

SAPA081: **Mathematical modelling with differential equations I**	**Credits:** **12**	**Co-requisite / Prerequisite:** **An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third year level Mathematics.**
Content: Nonlinear Systems: Basic Dynamical Systems Theory, Stability, and Bifurcation Analysis. Basic Theory of Differential Equations (ordinary differential equation (ODE), partial differential equation (PDE), stochastic differential equation (SDE), functional differential equation (FDE)), Flows and Maps. Reduction to Systems, Nonautonomous Systems, Phase Space and Portraits, Asymptotic Behavior, One-dimensional Flows and Maps, Poincare Map, Generation of Flow by an ODE, PDE, SDE and FDE. Contraction Mapping Principle of Banach-Cacciopoli, Existence and Uniqueness Theorems, Gronwall Lemma, Continuity in Initial Conditions, Two-dimensional Systems: Linear Systems, Hamiltonian and Integrable Systems, Poincare-Bendixson Theorem, Geometric Techniques: Stability and Attraction, Lasalle Invariance Principle. Invariant Manifolds, including Stable and Unstable Manifolds for Equilibria.		

SAPB081: **Stochastic differential equations**	**Credits: 12**	**Co-requisite / Prerequisite:** **An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third year level Mathematics.**
Content: Quick review of probability, stochastic processes and (local) martingales. Brownian motion and its properties, distribution of first hitting times, maximum and minimum, quadratic variation. Ito's calculus: stochastic integrals, Ito's formula, exponential martingales, Girsanov's theorem, the martingale representation theorem. Stochastic differential equations: weak and strong solutions, existence and uniqueness of solutions.		

SAPC081: **Advanced numerical analysis**	**Credits: 12**	**Co-requisite / Prerequisite:** **An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third year level Mathematics.**
Content: Iterative methods for solving linear systems: Jacobi and Gauss-Seidel method, and their analyses. Approximation theory: Weierstrass approximation theorem, Continuous Least Squares problem; solution using standard and orthogonal bases, Discrete Least Squares problem. Numerical integration using Gaussian quadrature, Multiple integrals. If time permits introduce Successive over-relaxation (SOR) and multi-grid methods for solving linear systems.		

SAPD081: **Advanced mathematical programming**	**Credits: 8**	**Co-requisite / Prerequisite:** **An average final mark of at least 60% in Applied**

		Mathematics on NQF Level 6 plus third year level Mathematics.
Content: Linear programming: Basic ideas and concepts of program formulation, Simplex method, Dual problem solution & its relation to the primal. Nonlinear programming (NLP) background involves classification of problems/programs according to: Minimization of unconstrained NLPs, Linearly constrained NLPs that include a special subclass of quadratic programs concerned with minimization of quadratic functions, Objective function having appropriate convexity property. Solution Methods: Lagrangian function with associated multipliers and conditions, Kuhn-Tucker conditions for inequality constrained minimization problems.		

SMTC081: General Topology	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Topological spaces, continuous functions, weak and strong topologies, quotient spaces, nets and filters, separation axioms, countability, compact spaces and connectedness.		

SMTD081: Complex Analysis	**Credits: 8**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Mobius Transformations and properties of Mobius transformaton, Uniform Convergence & Infinite Products; Entire Functions; Weierstrass' Theorem; Approximation by Rational Functions & Polynomials; Conformal Mappings; Riemann's Mapping Theorem		

SMTE081: Measure Theory and Integration	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Algebra of sets and limits; set functions. Outer and inner measure; extension of measure; - finite measures, Lebesque measure, Riemann-Stieltjies Measure, Measurable functions, Measure space, convergence in measure. Lebesque integrable functions, convergence in mean, properties in integrals. Countable additive set functions, Hahn and Jordan decomposition, Radon-Nikodyn theorem. Differentiation: Functions of bounded variation, Vitali's theorem.		

SMTG081: Group and Field Theory	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Groups: Review of group theory, examples of group, cyclic groups, permutation groups, groups up to order 16. Group actions and conjugacy classes. The isomorphism theorems; normality, the Jordan Holder theorem, solvable and simple groups. Direct products. Finitely Generated Abelian groups, free groups. Fields: Field extensions, splitting fields, Main Theorem of Galois, solution of polynomial Equations by radicals.		

SAPA082: **Mathematical modelling with differential equations II**	**Credits:** **12**	**Co-requisite / Prerequisite:** **An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third year level Mathematics.**
Content: Center Manifolds, Normal Forms and Bifurcation Theory for Equilibria including Poincare-Andronov-Hopf Bifurcations. Nonlinear Oscillations and the Method of Averaging. Smale Horseshoe Map, Smale-Birkhoff Theorem and Chaotic Transport. Melnikov Method and Homoclinic Bifurcation Theory. Geometric Singular Perturbation Theory, Shilnikov Orbits, Inclination-flip Homoclinic Orbits. Models From Beam Dynamics. Surface Wave Oscillations, Turbulent Boundary Layer Models, Pulse-propagation in Optical Fibers, Multi-degree-of-freedom Rigid Body Dynamics. Turbulence and transport; Wave-turbulence interactions in Geophysical and Astrophysical Fluid Dynamics; Topics in Atmospheric/Climate Dynamics and Related Themes; Chaos and Strange Attractors; Theoretical Aspects of Spiral and Scroll Wave Dynamics; Recent Applications of Geometric Singular Pertubation Theory; Dynamical Systems Aspects of Legged Locomotion.		

SAPB082: **Hydrodynamic Stability**	**Credits: 12**	**Co-requisite / Prerequisite:** **An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third year level Mathematics.**
Content:		

Linear stability and the basic procedure for analysing it. Thermal instabilities: linearised stability equations, solution using normal modes. Centrifugal instabilities: Rayleigh's criterion for instability. Shear flow instabilities: Orr-Somerfield equation, Squire theorem, Raleigh equation

SAPC082: **Financial Mathematics**	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third year level Mathematics.**
Content: Probability and events, conditional probability, random variables and expected values, covariance and correlation. Normal random variables and their properties, central limit theorem. Pricing models; geometric Brownian motion and its use in pricing models. Brownian motion. Interest rates and Present Value Analysis – including rate of return and continuously varying interest rates. Pricing contracts via arbitrage – options pricing and examples. The arbitrage theorem – proof and interpretation. The Black-Scholes Formula. Properties of the Black-Scholes option cost. Arbitrage strategy. A derivation of the Black-Scholes formula. Numerical binomial tree method.		

SAPD082: **Numerical solution of partial differential equations**	**Credits: 8**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Applied Mathematics on NQF Level 6 plus third**

		year level Mathematics.
Content: Finite difference approximations to derivatives. Parabolic equations: explicit and implicit schemes, Local truncation error, convergence of explicit scheme, stability analysis using the Von Neumann method. Hyperbolic equations: Upwind and Leapfrog schemes, characteristics, CFL condition, convergence, propagation of discontinuities. Elliptic equations: five point formula, finite differences in polar coordinates, finite difference formulae for curved boundaries.		

SMTA082: **Functional Analysis**	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Complete Metric spaces, Normed spaces and Banach spaces, Hahn-Banach Theorems, Open mapping Theorem and Closed graph Theorem, Linear operators and functionals, Dual spaces, Adjoint operators, Inner product and Hilbert spaces, Orthogonal complements and direct sums, Representation of functionals on Hilbert spaces, Hilbert-Adjoint operator, Finite Dimensional Normed Spaces, Spectral theory of bounded self-adjoint linear operators.		

SMTB082: Category Theory	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Definition of a category; Examples of categories drawn from different branches of mathematics; Faithful and full functors; Equivalence of categories. Special morphisms:		

Epimorphisms, monomorphisms and isomorphisms; Regular, split and strong monomorphisms and epimorphisms. Duality, contravariance and opposites; Products of categories; Functor categories; Comma categories. Cones and limits; Cocones and colimits; Examples of limits; Teminal objects, Products, Equalizers, Pullbacks and their duals; Construction of limits from products and equalizers

SMTD082: Number Theory	**Credits: 8**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Prime Number Theorem, Partition Function, Sequences with Arithmetic Progression, Sums & Differences, Products & Divisibility, Riemann Zeta Function		

SMTI082: Ring, Fields and Galios Theory	**Credits: 12**	**Co-requisite / Prerequisite: An average final mark of at least 60% in Mathematics on NQF Level 6**
Content: Axioms for Rings, Constructions for Rings, Quotient Rings, Integral Domains and Fields, The Field of Quotients, Polynomials, The Division Algorithm, Principal Ideal Domains, Unique Factorization, Prime fields, Euclidean algorithm, Commutative quotient rings, Quadratic and Cubic Equations, Algebraic and Transcendental Elements, Ruler and Compass, Algebraic extensions, Separable extensions, Normal extensions, Galois extensions, The Solution of Equations by Radicals.		

Master's programme in Mathematics

Dissertation (Mathematics)	**Credits: 180**	**Co-requisite / Prerequisite: An average final mark of at least 60% in the Honours degree in Mathematics**
Content: The Master of Science in Mathematics comprises a dissertation based on the research results of an approved supervised research project in the field of Mathematics, presented in a form suitable for publication.		

Master's programme in Applied Mathematics

Dissertation (Applied Mathematics)	**Credits: 180**	**Co-requisite / Prerequisite: An average final mark of at least 60% in the Honours degree in Applied Mathematics**
Content: The Master of Science in Applied Mathematics comprises a dissertation based on the research results of an approved supervised research project in the field of Applied Mathematics, presented in a form suitable for publication.		

Doctor of Philosophy in Applied Mathematics

THESIS (Applied Mathematics) SAMA100	**Credits: 360**	**Co-requisite / Prerequisite: The candidate should have an applicable Master of Science degree**
Content: The Doctor of Philosophy in Mathematics comprises a thesis based on the research results of an original, approved supervised research project in the field of Mathematics, presented in a form suitable for publication.		

Doctor of Philosophy in Mathematics

THESIS **(Mathematics)** **SMAA100**	**Credits: 360**	**Co-requisite / Prerequisite: The candidate should have an applicable Master of Science degree.**
Content: The Doctor of Philosophy in Applied Mathematics comprises a thesis based on the research results of an original, approved supervised research project in the field of Applied Mathematics, presented in a form suitable for publication.		